Решение.

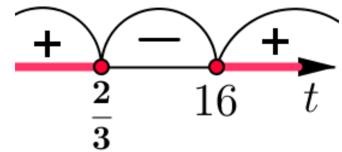
ОДЗ: $x+1>0 \iff x>-1$; перепишем неравенство в виде:

$$3\cdot(x+1)^{2\log_2(x+1)}-48\cdot\left(2^{\log_2(x+1)}\right)^{\log_2(x+1)}\geq 2\cdot(x+1)^{\log_2(x+1)}-32 \iff$$

$$\iff 3\cdot\left((x+1)^{\log_2(x+1)}\right)^2-48\cdot(x+1)^{\log_2(x+1)}\geq 2\cdot(x+1)^{\log_2(x+1)}-32.$$

Пусть $t=(x+1)^{\log_2(x+1)}, t>0,$ неравенство примет вид: $3t^2-48t\geq 2t-32 \iff$

$$\iff 3t(t-16)-2(t-16)\geq 0 \iff (t-16)(3t-2)\geq 0 \iff \left[\begin{array}{l} t\leq\frac{2}{3}, \\ t\geq 16. \end{array}\right.$$



Вернёмся к переменной x ($y=\log_2 t$ — возрастающая функция):

$$\left[\begin{array}{l} (x+1)^{\log_2(x+1)}\leq\frac{2}{3}, \\ (x+1)^{\log_2(x+1)}\geq 16; \end{array}\right. \iff \left[\begin{array}{l} \log_2(x+1)^{\log_2(x+1)}\leq\log_2\frac{2}{3}, \\ \log_2(x+1)^{\log_2(x+1)}\geq\log_2 16, \end{array}\right. \iff \left[\begin{array}{l} \log_2^2(x+1)\leq\log_2\frac{2}{3}\ \ (*), \\ \log_2^2(x+1)\geq 4; \end{array}\right. \iff$$

$(*)$ $\log_2\frac{2}{3}<0 \iff \log_2\frac{2}{3}<\log_2 1 \iff \frac{2}{3}<1$; левая часть неравенства неотрицательна, правая — отрицательна $\Rightarrow$ неравенство $(*)$ решений не имеет.

$$\iff |\log_2(x+1)|\geq 2 \iff \left[\begin{array}{l} \log_2(x+1)\geq 2, \\ \log_2(x+1)\leq -2; \end{array}\right. \iff \left[\begin{array}{l} \log_2(x+1)\geq\log_2 4, \\ \log_2(x+1)\leq\log_2 0{,}25; \end{array}\right. \iff \left[\begin{array}{l} x+1\geq 4, \\ 0<x+1\leq 0{,}25; \end{array}\right. \iff$$

$$\iff \left[\begin{array}{l} x+1\geq 4, \\ 0<x+1\leq 0{,}25; \end{array}\right. \iff \left[\begin{array}{l} x\geq 3, \\ -1<x\leq -0{,}75. \end{array}\right.$$

Ответ: $(-1;-0{,}75]\cup[3;\ +\infty)$.